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Acceptance sampling plan for multiple manufacturing lines using EWMA process capability index

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Abstract
The problem of developing a product acceptance determination procedure for multiple characteristics has attracted the quality assurance practitioners. Due to sufficient demands of consumers, it may not be possible to deliver the quantity ordered on time using the process based on one manufacturing line. So, in factories, product is manufactured using multiple manufacturing lines and combine it. In this manuscript, we present the designing of an acceptance sampling plan for products from multiple independent manufacturing lines using exponentially weighted moving average (EWMA) statistic of the process capability index. The plan parameters such as the sample size and the acceptance number will be determined by satisfying both the producer's and the consumer's risks. The efficiency of the proposed plan will be discussed over the existing sampling plan. The tables are given for industrial use and explained with the help of industrial examples. We conclude that the use of the proposed plan in these industries minimizes the cost and time of inspection. Smaller the sample size means low inspection cost. The proposed plan for some non-normal distributions can be extended as a future research. The determination of sampling plan using cost model is also interested area for the future research.

Key words : Sampling plan, Critical acceptance number, Producer's risk, Normal distribution, Consumer's risk

1. Introduction

In supply chain management, the role of product acceptance determination is very important. It addresses the problem of accepting or rejecting the manufactured product based on the information obtained for the inspection data. The product acceptance plan works under the designated risks specified by the producer and consumer [Pearn et al., 2013a]. For product acceptance determination, it is common to inspect a few items from the finished submitted lot for disposition of the lot. Therefore, there is chance for rejecting a good lot and accepting a bad lot. The chance of rejecting a good lot is termed as "producer's risk" and the chance of accepting a bad lot is called "consumer's risk". Therefore, the product acceptance determination using a sampling plan faces these two risks. In a sampling plan, the plan parameters are determined under the designated risks using the operating characteristics (OC) curve.

There are many sampling schemes which have been widely used for the inspection of the submitted lot of product. The single sampling plan is simplest and popularly used in practice. A sample is selected from the lot and number of defective is counted. A lot of product is accepted if number of non-conforming items is less than the specified number of failures. Otherwise, lot is rejected. A sampling plan is said to be more efficient if it provides the smaller sample size as compared to existing sampling plan. Several authors proposed various sampling plans for various situations including for example, Yen et al. (2014) designed sampling plan using EWMA yield index. Aslam et al. (2013a) worked for resubmitted sampling plan using process capability index. Aslam et al. (2015) designed SkSP-V sampling plan using process capability index. Yen et al. (2015) studied repetitive sampling plan for one-sided specification. Jun et al. (2014) proposed mixed multiple dependent state sampling plan using process capability index.

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Several authors worked on the designing of product determination plans for various distributions including for example Pearn and Wu (2006a) developed the product determination procedure when the quality of interest follows the normal distribution having one specification limit. Pearn and Wu (2007) extended the work of Pearn and Wu (2007) for two specification limits. Later on, Pearn and Wu (2006b) and Wu and Pearn (2008) proposed the product acceptance determination procedures for low fraction defective products. Itay et al. (2009) and Negrin et al. (2011) developed the multi-stage sampling plan for normal distribution. Pearn et al. (2013b) proposed the extended plan for multiple characteristics. More details about sampling schemes using process capability index can be seen in Aslam et al. (2013b), Pearn and Wu (2013), Alaeddini et al. (2009), Nezhad and Niaki (2010) and Aslam et al. (2014).

Most sampling plans in the literature use only the current information to make the final decision about the submitted lot of the product. This type of product acceptance determination plan is called "*memoryless"* product acceptance determination procedure. The efficiency of the plan can be increased by utilizing the current as well as the past information about the disposition of the submitted lot of the product. The exponentially weighted moving average (EWMA) statistic is one of the widely statistic in the area of control charts. As mentioned by Lucas and Saccucci (1990) and Montgomery (2007), the EWMA statistic gives high weight to the current information and decreasing weight to previous information. According to Čisar and Čisar (2011) "the EWMA is a statistic for monitoring the process that averages the data in a way that gives less and less weight to data as they are further removed in time."

The problem of developing a product acceptance determination procedure for multiple characteristics has attracted the quality assurance practitioners. Due to sufficient demands of consumers, it may not be possible to deliver the quantity ordered on time using the process based on one manufacturing line. So, in factories, product is manufactured using multiple manufacturing lines and combine it. For example, thin-film transistor type liquid-crystal display (TFT-LCD) glass is manufactured by using multiple independent manufacturing lines [Pearn et al. (2013a]. It is important to note here that the mean and variance of each line may be different and the combined output of all lines makes the decision about yield measurement difficult. Recently, Pearn et al. (2013a) designed a product determination plan for multiple independent lines.

The main objective of this paper is to design a new acceptance sampling plan for product from multiple independent manufacturing lines. The use of EWMA statistic of the process capability index is proposed for multiple independent manufacturing lines to reduce the sample size required for the acceptance sampling plan. The structure of the proposed plan will be given. The advantage of the proposed plan over Pearn et al. (2013a) will be discussed. The proposed plan will be explained with the help of examples.

2 Designing of Proposed Plan

According to Pearn et al. (2013a) "manufacturing process with multiple manufacturing lines often consists of multiple parallel independent manufacturing lines, with each manufacturing line having a machine or a group of machines performing necessary identical job operations. As the manufacturing lines have various process averages and standard deviations, the values of capability indices will be different for each manufacturing line. The combined output of all manufacturing lines leads to inaccurate yield measures of the process".

Tai et al. (2012) proposed the following overall capability index:

$$\mathrm{S}_{\mathrm{pk}}^{\mathrm{M}} = \frac{1}{3}\Phi^{-1}\left\{\left[\frac{1}{\mathrm{k}}\sum_{\mathrm{j}=1}^{\mathrm{k}}\left(2\Phi\left(3\mathrm{S}_{\mathrm{pkj}}\right)-1\right)+1\right]/2\right\} \quad (1)$$

where $\mathrm{S}_{\mathrm{pkj}}$ denotes the S_{pk} value of j^{th} line for $j = 1,2,\dots,k$, k is number of manufacturing lines, S_{pk} is traditional process capability index and $\Phi(.)$ is the cumulative distribution function (cdf) of a standard normal distribution. Note here that the statistic $\mathrm{S}_{\mathrm{pk}}^{\mathrm{M}}$ proposed by Tai et al. (2012) is an extension of the statistic proposed by Boyles (1994). The means and variances of multiple independents lines are unknown in practice and estimated using the sample data, therefore, estimate of $\mathrm{S}_{\mathrm{pk}}^{\mathrm{M}}$ is given as follows

$$\hat{\mathrm{S}}_{\mathrm{pk}}^{\mathrm{M}} = \frac{1}{3}\Phi^{-1}\left\{\left[\frac{1}{\mathrm{k}}\sum_{\mathrm{j}=1}^{\mathrm{k}}\left(2\Phi\left(3\hat{\mathrm{S}}_{\mathrm{pkj}}\right)-1\right)+1\right]/2\right\} \quad (2)$$

where

$$\hat{S}_{pkj} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL-\bar{X}_j}{s_j}\right)+\frac{1}{2}\Phi\left(\frac{\bar{X}_j-LSL}{s_j}\right)\right\} \quad (3)$$

Note that USL and LSL denote the upper specification limit and lower specification limit, respectively.

Where $\bar{X}_j$ and S_j are the mean and standard deviation of j^{th} manufacturing line. Tai et al. (2012) used the Taylor expansion method and derive the following asymptotic normal distribution of $\hat{S}_{pk}^{M}$

$$\hat{S}_{pk}^{M} \sim N\left(S_{pk}^{M}, \frac{1}{36k^2 n\left[\Phi\left(3S_{pk}^{M}\right)\right]^2}\sum_{j=1}^{k}\left(a_j^2 + b_j^2\right)\right)$$

where

$$a_j = \left\{\left[\frac{USL-\mu_j}{\sqrt{2}\sigma_j}\right]\Phi\left[\frac{USL-\mu_j}{\sigma_j}\right] + \left[\frac{\mu_j - LSL}{\sqrt{2}\sigma_j}\right]\Phi\left[\frac{\mu_j - LSL}{\sigma_j}\right]\right\}$$

$$b_j = \left\{\Phi\left[\frac{USL-\mu_j}{\sigma_j}\right] + \Phi\left[\frac{\mu_j - LSL}{\sigma_j}\right]\right\}$$

where Φ shows the probability density function of normal distribution, $N(.)$ denotes normal distribution, μ_j and σ_j are mean and standard deviation of j^{th} manufacturing line and a_j and b_j are the parameters for j^{th} manufacturing line. Tai et al. (2012) provided the following simplified form of asymptomatic distribution, see [Pearn et al. (2013a)]

$$\hat{S}_{pk}^{M} \sim N\left(S_{pk}^{M}, \frac{D^2\phi^2(3D)}{2k^2 n\phi^2\left(3S_{pk}^{M}\right)}\right)$$

where

$$D = (1/3)\Phi^{-1}\left\{\left[k\left(2\Phi\left(3S_{pk}^{M}\right)-1\right)-(k-2)\right]/2\right\} \quad (4)$$

where Φ shows cdf of the standard normal distribution.

The proposed plan based on EWMA of the overall process capability index is stated as follows:

Step-1: Select a random sample of size $n_j = n, j = 1,2,3,\dots,k$, from line j at time i. Calculate the process capability index in (3) for each multiple line S_j at time i. Compute the overall index $\hat{S}_{pk}^{M}$ as follows:

$$\hat{S}_{pk}^{M} = \frac{1}{3}\Phi^{-1}\left\{\left[\frac{1}{k}\sum_{j=1}^{k}\left(2\Phi\left(3\hat{S}_{pkj}\right)-1\right)+1\right]/2\right\}$$

Compute the following EWMA statistic at time i:

$$\hat{S}_{pk}^{M\,EWMA_i} = \lambda\hat{S}_{pk}^{M} + (1-\lambda)\hat{S}_{pk}^{M\,EWMA_{i-1}}$$

where λ is a smoothing constant and ranges from 0 and 1.

Step-2: Accept the lot if $\hat{S}_{pk}^{M\,EWMA_i} \geq c$ otherwise; reject the lot, where is c is the critical acceptance number.

The proposed sampling plan is the extension of plan given by Pearn et al. (2013a). The proposed plan is characterized for two parameters namely n_j and c for other specified parameters. The proposed plan utilizes current information and past information using λ to make decision about the submitted lot of product, while Pearn et al. (2013a) plan utilizes only current information to make decision about the submitted lot. The proposed plan reduces to Pearn et al. (2013a) sampling plan when $\lambda = 1$. Now, we derive the OC function of the proposed plan as follows.
According to the plan, the lot of product will be accepted if

$$P\left(\hat{S}_{pk}^{M\,EWMA_i} \geq c\right) = P\left(\frac{\hat{S}_{pk}^{M\,EWMA_i} - S_{pk}^{M}}{\sqrt{(\lambda/(2-\lambda))\left[\frac{D^2\phi^2(3D)}{2k^2 n\phi^2\left(3S_{pk}^{M}\right)}\right]}} \geq \frac{c - S_{pk}^{M}}{\sqrt{(\lambda/(2-\lambda))\left[\frac{D^2\phi^2(3D)}{2k^2 n\phi^2\left(3S_{pk}^{M}\right)}\right]}}\right) \quad (5)$$

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Let $\frac{\hat{S}_{pk}^{M\,EWMA_i}-S_{pk}^{M}}{\sqrt{(\lambda/(2-\lambda))\left[\frac{D^2\phi^2(3D)}{2k^2n\phi^2\left(3S_{pk}^{M}\right)}\right]}}=z$, where is z is standard normal random variable, using it Eq. (5) can be rewritten as follows

$$P\left(\hat{S}_{pk}^{M\,EWMA_i}\geq c\right)=1-P\left(Z<\frac{c-S_{pk}^{M}}{\sqrt{(\lambda/(2-\lambda))\left[\frac{D^2\phi^2(3D)}{2k^2n\phi^2\left(3S_{pk}^{M}\right)}\right]}}\right) \tag{6}$$

Finally, the lot acceptance probability, say $\pi_A\left(\hat{S}_{pk}^{M\,EWMA_i}\right)$ is given as

$$\pi_A\left(\hat{S}_{pk}^{M\,EWMA_i}\right)=1-\Phi\left(\frac{c-S_{pk}^{M}}{\sqrt{(\lambda/(2-\lambda))\left[\frac{D^2\phi^2(3D)}{2k^2n\phi^2\left(3S_{pk}^{M}\right)}\right]}}\right) \tag{7}$$

Let α be the producer's risk and β be the consumer's risk. The sampling plan will be designated such that the product acceptance of good lot will be greater than producer's confidence level, say $1-\alpha$ at acceptable quality level (AQL) and the product acceptance of bad lot will be smaller than β at lot percent defective quality level (LTPD). Let $(AQL, 1-\alpha)$ and $(LTPD, \beta)$ are two points through the OC curve. The plan parameters of the proposed plan will be determined through following non-linear equations

$$P\{\text{Reject the lot}|\text{p}\leq\text{AQL}\}=P\left\{\hat{S}_{pk}^{M\,EWMA_i}<c|\hat{S}_{pk}^{M\,EWMA_i}\geq C_{AQL}\right\} \tag{8}$$

$$P\{\text{accept the lot}|\text{p}\geq\text{LTPD}\}=P\left\{\hat{S}_{pk}^{M\,EWMA_i}<c|\hat{S}_{pk}^{M\,EWMA_i}\leq C_{LTPD}\right\} \tag{9}$$

where C_{AQL} and C_{LTPD} capability value corresponding to AQL and LTPD on the basis of $\hat{S}_{pk}^{M\,EWMA_i}$ index. The complete non-linear solution is given as follows

Minimize n (10a)
Subject to

$$1-\Phi\left(\frac{c-C_{AQL}}{\sqrt{(\lambda/(2-\lambda))\left[\frac{D^2\phi^2(3D)}{2k^2n\phi^2\left(3S_{pk}^{M}\right)}\right]}}\right)\geq 1-\alpha \tag{10b}$$

$$1-\Phi\left(\frac{c-C_{LTPD}}{\sqrt{(\lambda/(2-\lambda))\left[\frac{D^2\phi^2(3D)}{2k^2n\phi^2\left(3S_{pk}^{M}\right)}\right]}}\right)\leq \beta \tag{10c}$$

The plan parameters of the proposed plan are determined for various values of α, β, C_{AQL}, C_{LTPD} and k are placed in Tables 1-4. From these tables, we note following trends in plan parameters

1. When α or β are large, the smaller the sample size for the inspection of the product is required. It means that quality level preset by both parties is relatively loose.
2. When α or β are small, the larger the sample size for the inspection of the product is required. It means that quality level preset by both parties is relatively high.
3. For other specified parameters, when multiple manufacturing lines are increasing, the smaller the sample size for the inspection of the product is required. As the proposed plan utilizes past information, for larger k is

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larger, the auditor has more past information and current information. So, smaller sample may need to make decision about the submitted lot.

4. For other specified parameters, when smoothing constant increasing, the larger the sample size for the inspection of the product is required. The value of λ should not be zero, which case cannot reflect the present state. When the process is very stable, a smaller value is preferred, but it may not reflect the sudden change in the process. So usually, the value between 0.1 and 0.3 is recommended to use.

For practical use of the proposed plan, following is step-by-step procedure to determine the plan parameters.

Step-1: preset the combination of (C_{AQL}, C_{LTPD}) and (α, β)

Step-2: preset the smoothing constant λ

Step-3: Check tables and select the corresponding values of plan parameters

Step-4: Determine $\hat{S}_{pk}^{MEWMA_i}$ using sample data

Step-5: Accept or reject the lot according to given decision criteria

Table 1 Plan parameters when $k = 2$ and $\lambda = 0.1$

C_{AQL}	C_{LTPD}	$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.01$		$\alpha = 0.025$		α=0.05		$\alpha = 0.1$	
		$\beta = 0.05$		$\beta = 0.075$		$\beta = 0.1$		$\beta = 0.125$		$\beta = 0.1$		$\beta = 0.1$		$\beta = 0.1$		$\beta = 0.1$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
1.00	0.90	6	0.9469	5	0.9442	5	0.9438	5	0.9367	8	0.9357	6	0.9352	5	0.9384	4	0.9452
1.05	0.95	7	0.9959	6	0.996	5	0.991	5	0.9894	9	0.9849	7	0.9908	5	0.9914	4	0.9989
1.10	1.00	8	1.044	7	1.0455	6	1.0426	6	1.0437	9	1.035	8	1.0379	6	1.0401	5	1.0494
1.15	1.05	8	1.0963	7	1.0941	7	1.0932	6	1.09	10	1.0844	8	1.0873	7	1.0903	5	1.0958
1.20	1.10	9	1.1464	8	1.1427	8	1.1425	7	1.1405	11	1.1326	9	1.1379	7	1.1425	6	1.1489
1.25	1.15	10	1.1962	9	1.194	8	1.1937	7	1.1883	13	1.1856	10	1.1886	8	1.1936	6	1.1992
1.30	1.20	11	1.2494	10	1.2423	9	1.2436	8	1.2406	13	1.2338	11	1.2382	9	1.2396	7	1.2481
1.35	1.25	12	1.3001	10	1.294	10	1.2901	9	1.2914	15	1.2855	11	1.2879	10	1.294	7	1.2972
1.40	1.30	12	1.348	11	1.3446	10	1.3424	10	1.3368	16	1.3359	12	1.3373	10	1.3421	8	1.3492
1.45	1.35	14	1.3973	12	1.3937	11	1.3932	10	1.3907	17	1.3826	13	1.3878	11	1.3929	8	1.3977
1.50	1.40	14	1.4482	13	1.4456	12	1.4409	11	1.4416	18	1.434	15	1.4388	12	1.4413	9	1.4475
1.55	1.45	16	1.4987	14	1.4957	13	1.4914	12	1.4879	19	1.4834	15	1.4876	13	1.4923	10	1.4982
1.60	1.50	17	1.5489	15	1.5449	13	1.5421	13	1.5394	20	1.5337	17	1.5396	13	1.5424	10	1.5481
1.65	1.55	18	1.5977	16	1.5948	15	1.5904	13	1.5901	22	1.5855	17	1.5879	15	1.5921	11	1.5971
1.70	1.60	19	1.6479	17	1.6447	15	1.6421	14	1.6408	23	1.6341	19	1.637	15	1.6427	12	1.6493
1.75	1.65	21	1.6991	18	1.6962	17	1.6911	15	1.6895	26	1.6843	20	1.6875	16	1.6918	13	1.6975
1.80	1.70	21	1.7485	19	1.7456	17	1.743	16	1.7404	26	1.7338	21	1.7378	18	1.7426	13	1.7477
1.85	1.75	23	1.7985	20	1.7953	18	1.793	17	1.7912	28	1.7844	23	1.7883	18	1.7919	15	1.7983
1.90	1.80	24	1.8482	22	1.844	20	1.841	18	1.8413	30	1.8351	24	1.8385	19	1.8426	15	1.8489
1.95	1.85	25	1.8984	23	1.8959	21	1.8943	18	1.8899	31	1.8843	25	1.889	21	1.8931	16	1.8985
2.00	1.90	27	1.9479	24	1.9449	22	1.9429	20	1.9413	33	1.9352	27	1.9392	21	1.9425	17	1.9501
1.00	0.85	3	0.9219	3	0.9039	2	0.9107	2	0.9036	3	0.8962	3	0.9058	2	0.908	2	0.9145
1.05	0.90	3	0.9638	3	0.9697	3	0.9688	2	0.9555	4	0.9506	3	0.9538	3	0.9662	2	0.9761
1.10	0.95	3	1.0195	3	1.0116	3	1.0164	3	1.02	4	1.0023	3	1.0034	3	1.0141	2	1.0224
1.15	1.00	4	1.0686	3	1.0659	3	1.0599	3	1.0526	5	1.0532	4	1.063	3	1.0617	2	1.0688
1.20	1.05	4	1.123	4	1.1188	3	1.1089	3	1.1031	5	1.0973	4	1.1093	3	1.1121	3	1.121

Table 2 Plan parameters when $k = 3$ and $\lambda = 0.1$

C_{AQL}	C_{LTPD}	$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.01$		$\alpha = 0.025$		α=0.05		$\alpha = 0.1$	
		$\beta = 0.05$		$\beta = 0.075$		$\beta = 0.1$		$\beta = 0.125$		$\beta = 0.1$		$\beta = 0.1$		$\beta = 0.1$		$\beta = 0.1$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
1.00	0.90	3	0.947	2	0.9435	2	0.9438	2	0.9428	3	0.9316	3	0.9396	2	0.9383	2	0.956
1.05	0.95	3	0.9991	3	0.9928	2	0.9911	2	0.9894	4	0.9803	3	0.9834	2	0.9907	2	1.0007
1.10	1.00	3	1.0462	3	1.0485	3	1.0422	3	1.041	4	1.0316	3	1.0367	3	1.0363	2	1.0448
1.15	1.05	4	1.1025	3	1.0947	3	1.0957	3	1.0931	5	1.084	4	1.0872	3	1.0949	2	1.0964
1.20	1.10	4	1.1498	4	1.147	3	1.1409	3	1.1381	5	1.132	4	1.1411	3	1.1424	3	1.1449
1.25	1.15	4	1.1972	4	1.1914	4	1.1893	3	1.1901	5	1.183	4	1.1873	4	1.1875	3	1.2026
1.30	1.20	5	1.2445	4	1.2459	4	1.2401	4	1.2422	6	1.2378	5	1.2372	4	1.2403	3	1.2479
1.35	1.25	5	1.2991	5	1.2974	4	1.2911	4	1.2871	6	1.2844	6	1.2912	4	1.2934	3	1.2971
1.40	1.30	6	1.3509	5	1.3454	5	1.3473	4	1.3386	7	1.3337	6	1.335	5	1.3455	4	1.3476
1.45	1.35	6	1.3959	5	1.3955	5	1.3933	5	1.3872	7	1.3845	6	1.389	5	1.3926	4	1.4025
1.50	1.40	7	1.4498	6	1.4437	5	1.4417	5	1.4417	8	1.4355	6	1.4383	5	1.4427	4	1.4464
1.55	1.45	7	1.4966	6	1.4962	6	1.4957	5	1.4885	9	1.488	7	1.4896	6	1.4894	5	1.4952
1.60	1.50	7	1.5482	7	1.5454	6	1.5412	6	1.5374	9	1.536	7	1.5376	6	1.543	5	1.5448
1.65	1.55	8	1.5985	7	1.595	6	1.5927	6	1.5892	10	1.5836	8	1.5869	7	1.5948	5	1.6007
1.70	1.60	8	1.6482	8	1.6473	7	1.6425	7	1.6403	10	1.6335	8	1.6382	7	1.6428	5	1.6476
1.75	1.65	9	1.6988	8	1.6954	7	1.6926	7	1.6879	11	1.683	9	1.6883	7	1.6929	6	1.6965
1.80	1.70	10	1.7491	8	1.7449	8	1.7454	7	1.7414	12	1.7332	10	1.7419	8	1.7447	6	1.7475
1.85	1.75	10	1.7993	9	1.7944	8	1.7938	8	1.7884	12	1.7837	10	1.7877	8	1.7937	6	1.798
1.90	1.80	11	1.848	9	1.8453	9	1.8401	8	1.8394	13	1.8345	11	1.841	9	1.8428	7	1.8474
1.95	1.85	11	1.8993	10	1.8958	10	1.893	8	1.8897	14	1.8862	11	1.8879	9	1.8928	7	1.8974
2.00	1.90	12	1.9473	11	1.9437	11	1.9466	9	1.9418	14	1.9347	12	1.9392	10	1.9414	7	1.9484
1.00	0.85	2	0.9323	2	0.9215	2	0.9425	2	0.8873	2	0.898	2	0.9335	2	0.8913	2	0.9035
1.05	0.90	2	0.9626	2	0.9572	2	0.9768	2	0.9686	2	0.9433	2	0.9407	2	0.9763	2	0.9788
1.10	0.95	2	1.0307	2	1.0028	2	1.0238	2	0.9999	2	0.9997	2	1.0212	2	1.0134	2	1.0503
1.15	1.00	2	1.0603	2	1.0791	2	1.0448	2	1.0413	2	1.0541	2	1.0611	2	1.0667	2	1.0636
1.20	1.05	2	1.1094	2	1.1212	2	1.1279	2	1.1258	2	1.0993	2	1.0996	2	1.1283	2	1.1102

Table 3: Plan parameters when $k = 2$ and $\lambda = 0.2$

C_{AQL}	C_{LTPD}	$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.01$		$\alpha = 0.025$		α=0.05		$\alpha = 0.1$	
		$\beta = 0.05$		$\beta = 0.075$		$\beta = 0.1$		$\beta = 0.125$		$\beta = 0.1$		$\beta = 0.1$		$\beta = 0.1$		$\beta = 0.1$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
1.00	0.90	12	0.9457	11	0.9428	10	0.9427	9	0.9397	15	0.934	13	0.9392	10	0.9422	8	0.9475
1.05	0.95	14	0.9966	13	0.9961	12	0.9942	10	0.9884	17	0.9823	13	0.9867	11	0.9902	9	0.9974
1.10	1.00	16	1.046	13	1.0443	12	1.0412	11	1.0386	19	1.0328	15	1.0378	12	1.0412	10	1.0453
1.15	1.05	17	1.0987	15	1.0942	14	1.0935	13	1.0909	20	1.0832	17	1.0883	14	1.0901	11	1.0992
1.20	1.10	18	1.1472	16	1.1448	15	1.1417	14	1.1375	23	1.1341	19	1.1374	15	1.1427	11	1.1476
1.25	1.15	20	1.1977	18	1.1936	16	1.1916	15	1.1885	26	1.1837	20	1.1873	16	1.1914	13	1.1988
1.30	1.20	22	1.2483	21	1.2443	18	1.2414	16	1.2394	27	1.2335	22	1.2376	18	1.2416	14	1.2465
1.35	1.25	25	1.2974	21	1.2941	20	1.293	18	1.2896	29	1.2835	24	1.2868	19	1.2916	15	1.2984
1.40	1.30	26	1.3479	23	1.3449	21	1.342	20	1.3401	32	1.3347	25	1.3377	21	1.3414	16	1.347
1.45	1.35	28	1.3988	25	1.394	24	1.3904	22	1.3887	34	1.384	28	1.3874	23	1.3923	18	1.399
1.50	1.40	31	1.4477	27	1.4448	25	1.4433	22	1.4396	37	1.4343	30	1.4384	24	1.4418	18	1.4481
1.55	1.45	32	1.4979	28	1.4948	27	1.4938	25	1.4883	40	1.4839	33	1.4876	26	1.4927	20	1.499

1.60	1.50	35	1.5485	31	1.5444	29	1.5416	26	1.5395	43	1.5342	35	1.5372	28	1.5423	21	1.5485
1.65	1.55	37	1.5988	33	1.5951	29	1.5922	27	1.5899	45	1.5842	37	1.5887	30	1.5916	23	1.5991
1.70	1.60	40	1.6483	35	1.6453	32	1.6424	29	1.6399	50	1.6334	39	1.6386	32	1.642	25	1.65
1.75	1.65	43	1.6995	38	1.6951	34	1.6918	31	1.6902	51	1.6841	43	1.6898	35	1.6922	26	1.6985
1.80	1.70	45	1.7492	40	1.7458	36	1.742	34	1.7389	55	1.734	45	1.7387	38	1.7437	27	1.7483
1.85	1.75	47	1.7985	43	1.7943	38	1.7919	34	1.7898	59	1.784	48	1.7879	38	1.7929	30	1.7993
1.90	1.80	52	1.8479	44	1.8452	40	1.8426	38	1.8393	61	1.8344	50	1.8392	41	1.8418	30	1.8486
1.95	1.85	53	1.8982	46	1.8953	43	1.8931	39	1.8896	67	1.8854	53	1.8885	44	1.8926	33	1.8976
2.00	1.90	56	1.9484	51	1.9456	45	1.9427	41	1.94	70	1.9354	55	1.9383	47	1.9425	34	1.9486
1.00	0.85	5	0.9187	5	0.9131	4	0.9094	4	0.9089	7	0.8984	5	0.9018	4	0.9092	3	0.9165
1.05	0.90	6	0.9665	5	0.9645	5	0.9571	5	0.9576	7	0.9479	6	0.9537	5	0.9589	4	0.9628
1.10	0.95	7	1.0143	6	1.0121	6	1.0159	5	1.0055	8	0.998	7	1.0006	6	1.0116	4	1.0198
1.15	1.00	7	1.0684	7	1.0659	6	1.0618	6	1.0516	9	1.0499	7	1.0541	6	1.0612	5	1.0662
1.20	1.05	8	1.1177	7	1.1134	7	1.1076	6	1.1062	10	1.1012	8	1.1055	7	1.1165	6	1.1163

Table 4 Plan parameters when $k = 3$ and $\lambda = 0.2$

C_{AQL}	C_{LTPD}	$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.05$		$\alpha = 0.01$		$\alpha = 0.025$		α=0.05		$\alpha = 0.1$	
		$\beta = 0.05$		$\beta = 0.075$		$\beta = 0.1$		$\beta = 0.125$		$\beta = 0.1$		$\beta = 0.1$		$\beta = 0.1$		$\beta = 0.1$	
		n	c	n	c	n	c	n	c	n	c	n	c	n	c	n	c
1.00	0.90	5	0.9458	5	0.9443	4	0.9419	4	0.9378	6	0.9318	5	0.9348	4	0.9429	3	0.9488
1.05	0.95	6	0.9985	5	0.9954	5	0.989	4	0.9891	7	0.9827	6	0.986	5	0.9901	4	0.9959
1.10	1.00	6	1.0473	6	1.0432	5	1.0403	5	1.0402	8	1.0356	7	1.0342	5	1.0425	4	1.046
1.15	1.05	7	1.0965	6	1.0938	6	1.0935	5	1.0884	9	1.0861	7	1.0876	6	1.0934	4	1.0972
1.20	1.10	8	1.1481	7	1.1458	6	1.1417	6	1.1416	10	1.1354	8	1.137	6	1.1412	5	1.149
1.25	1.15	9	1.1987	8	1.1957	7	1.1912	7	1.1884	11	1.1826	9	1.188	7	1.1931	6	1.1955
1.30	1.20	10	1.2505	8	1.2441	8	1.2431	7	1.2398	12	1.2356	10	1.2362	8	1.2427	6	1.2458
1.35	1.25	10	1.2973	10	1.2962	9	1.2948	8	1.2887	13	1.2841	10	1.2871	8	1.2916	7	1.3017
1.40	1.30	12	1.3493	10	1.346	9	1.3424	8	1.339	14	1.3347	11	1.337	9	1.3424	7	1.3481
1.45	1.35	13	1.399	11	1.3936	10	1.3916	9	1.3909	15	1.385	12	1.3886	10	1.3912	8	1.3993
1.50	1.40	13	1.4475	12	1.4448	11	1.4443	10	1.4392	16	1.4331	13	1.4367	11	1.4437	8	1.4489
1.55	1.45	15	1.4971	13	1.4935	11	1.4922	10	1.4891	17	1.4835	14	1.4875	11	1.4922	9	1.4961
1.60	1.50	15	1.5487	14	1.5436	13	1.5432	11	1.539	19	1.5335	15	1.5383	12	1.5426	9	1.5485
1.65	1.55	16	1.5988	15	1.595	14	1.5924	12	1.5909	20	1.5844	16	1.5885	13	1.5923	10	1.598
1.70	1.60	17	1.6484	16	1.6449	14	1.6427	13	1.6393	21	1.6338	17	1.6373	14	1.643	11	1.6466
1.75	1.65	19	1.7	16	1.695	15	1.692	14	1.6891	22	1.684	18	1.6878	16	1.6929	11	1.698
1.80	1.70	20	1.7496	18	1.7445	16	1.7434	16	1.7422	24	1.7347	21	1.741	16	1.7427	12	1.7489
1.85	1.75	21	1.7984	19	1.7956	17	1.7921	15	1.7898	25	1.7846	21	1.7889	17	1.7914	13	1.7973
1.90	1.80	22	1.8478	20	1.845	18	1.8434	16	1.8402	28	1.8361	22	1.8383	18	1.8434	14	1.8499
1.95	1.85	24	1.8984	21	1.8958	19	1.8935	18	1.8893	29	1.8844	23	1.8882	19	1.8913	15	1.8997
2.00	1.90	25	1.9482	22	1.946	21	1.9421	18	1.9393	30	1.9345	24	1.938	20	1.9414	16	1.9498
1.00	0.85	2	0.9183	2	0.9193	2	0.9049	2	0.9164	3	0.9001	2	0.9033	2	0.919	2	0.9369
1.05	0.90	3	0.9749	2	0.9617	2	0.9561	2	0.9561	3	0.9499	3	0.9557	2	0.9593	2	0.9791
1.10	0.95	3	1.0239	3	1.0256	2	1.0092	2	1.0093	4	0.9946	3	1.002	2	1.0093	2	1.0195
1.15	1.00	3	1.0665	3	1.0702	3	1.0654	3	1.0641	4	1.0492	3	1.0527	3	1.0653	2	1.0628
1.20	1.05	4	1.1193	3	1.1114	3	1.1177	3	1.1125	5	1.0995	4	1.1002	3	1.1156	2	1.1189

3. Advantages of Proposed Plan

In this section, the advantages of the proposed plan over the Pearn et al. (2013a) plan will be given. To compare the efficiency of the proposed plan over Pearn et al. (2013a) plan, the same values of all parameters are set.

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The plan parameters of the both sampling plans are placed in Table 5.

Table 5 Comparison of Sample Size for Proposed and Existing Plans

C_{AQL}	C_{LTPD}	$\lambda = 0.1$				$\lambda = 0.2$			
		Existing Plan		Proposed $k = 2$		Existing Plan		Proposed $k = 2$	
		$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.1$
		$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.05$	$\beta = 0.1$
		n	n	n	n	n	n	n	n
1.05	0.95	30	19	7	4	61	37	14	9
1.10	1.00	33	21	8	5	70	42	16	10
1.15	1.05	35	22	8	5	75	47	17	11
1.20	1.10	38	23	9	6	81	50	18	11
1.25	1.15	42	26	10	6	87	54	20	13
1.30	1.20	47	28	11	7	97	59	22	14
1.35	1.25	49	30	12	7	105	63	25	15
1.40	1.30	53	32	12	8	111	67	26	16
1.45	1.35	58	36	14	8	119	74	28	18
1.50	1.40	62	37	14	9	129	78	31	18
1.55	1.45	65	40	16	10	136	84	32	20
1.60	1.50	70	43	17	10	146	90	35	21
1.65	1.55	74	46	18	11	158	96	37	23
1.70	1.60	79	50	19	12	167	100	40	25
1.75	1.65	84	51	21	13	175	109	43	26
1.80	1.70	89	55	21	13	191	116	45	27
1.85	1.75	93	57	23	15	197	120	47	30
1.90	1.80	98	61	24	15	206	126	52	30
1.95	1.85	106	63	25	16	224	133	53	33
2.00	1.90	110	66	27	17	230	141	56	34
1.00	0.85	11	7	3	2	24	15	5	3
1.05	0.90	14	8	3	2	26	16	6	4
1.10	0.95	14	9	3	2	29	18	7	4
1.15	1.00	15	10	4	2	31	19	7	5
1.20	1.05	17	10	4	3	35	21	8	6

From Table 5, it is clear that the proposed plan has advantage in providing the smaller values of sample as compared to (Pearn et al., 2013a) sampling plan for all specified parameters. For example, when $= 0.1,\ k = 2, \alpha = 5\%, \beta = 5\%,\ C_{AQL}$=1.05 and C_{LTPD}=0.95, the sample size required for the inspection of lot is 7 from the proposed plan and it is 30 from the (Pearn et al., 2013a) sampling plan. It means that the proposed plan brings about 4 times reduction in the sample size required for the inspection of the lot of the product. Similarly, when $= 0.2,\ k = 2,\ \alpha = 5\%, \beta = 5\%,\ C_{AQL}$=1.05 and C_{LTPD}=0.95, the sample size required for the inspection of lot is 14 from the proposed plan and it is 61 from the (Pearn et al., 2013a) sampling plan. It means that the proposed plan brings 4 times reduction in the sample size required for the inspection of the lot of the product.

4 Industrial Examples

4.1 Application in Gold Bumping Process

For the application of the proposed plan in the inspection of gold bumping process, we will consider the product made by factory located the Science Based Industrial Park at Hsinchu, Taiwan [Pearn et al. (2013b)]. The quality engineer of IC design house applies the sampling plan for the inspection of production product FHD1080H (FHD, 1920×1080 RGB). This production come from three independent manufacturing lines and data is collected from each line separately. More details can be seen in Pearn et al. (2013b). The USL=10.5 μm, LSL=7.5 μm and target value is 9 μm. It is important to note that if the quality characteristic does not fall between in USL and LSL shows the

decreasing of product reliability.
The sample estimators from the data collected by three independent lines are given as follows Pearn et al. (2013b). The data is given in Table 6.

Table 6 Sample data

Lines	$\bar{X}_j$	S_j	$\hat{S}_{pkj}$
1	8.125	0.2027	1.0946
2	9.735	0.1351	1.9267
3	8.991	0.3286	1.5210

Using the above sample information, the calculated value of $\hat{S}_{pk}^{M} = 1.1936$. Let $\lambda = 0.1$, $k = 3$,$C_{AQL} = 1.20$, C_{LTPD}=1.10, $\alpha = 0.01$ and $\beta = 0.1$ The required sample size for the inspection of FHD1080H product is 48 from Pearn et al. (2013b) sampling plan while it is only 10 from the propsed sampling plan. The proposed plan is more economical than Pearn et al. (2013b) sampling plan for the inspection of FHD1080H product.

4.2 Application in TFT-LCD Inspection

As mentioned earlier that TFT-LCD process is multiple manufacturing processes which is widely used in cell phones, personal digital assistants, notebooks computer and monitors. For more details, see [Pearn et al. (2013a)]. Suppose that the production came from three independent and normal distributed lines. The USL=0.77 mm, LSL=0.63 mm and target value is 0.70 mm.
The sample estimator from the data collected by three independent lines are given as follows [Pearn et al. (2013a)]. The data is given in Table 7.

Tabel 7 Sample data

Lines	$\bar{X}_j$	S_j	$\hat{S}_{pkj}$
1	0.7211	0.0115	1.46829
2	0.6984	0.0097	2.37856
3	0.7029	0.0023	1.85550

Using the above sample information, the calculated value of $\hat{S}_{pk}^{M} = 1.545626$. Let $\lambda = 0.1$, $k = 3$,$C_{AQL} = 1.50$, C_{LTPD}=1.30, $\alpha = 0.05$ and $\beta = 0.10$ The required sample size for the inspection of TFT-LCD product is 169 from (Pearn et al., 2013) sampling plan while it is only 2 from the propsed sampling plan. The proposed plan is more economical than Pearn et al. (2013) sampling plan for the inspection of TFT-LCD product.

4.3 Results and Discussion

As the testing/inspection cost is directly related to the sample size selected for inspection of lot. The efficiency of any plan can be compared with any other plan in terms of sample size using the same specified parameters. A sampling plan which provides a smaller sample size for lot inspection is said to be more efficient plan. By comparing the proposed plan with the existing sampling plan, we note that the proposed plan provides much reduction in sample size for inspection. Therefore, the use of the proposed plan for inspection of submitted lots will be more economical for industry.

5 Concluding Remarks

In this manuscript, a sampling plan for multiple lines is proposed using the EWMA statistic. The plan parameters are determined under the various designated producer's and consumer's risks. The efficiency of the proposed sampling is discussed over the existing sampling plan. We conclude that the proposed plan is more efficient than the existing sampling plan in terms of sample size required for the inspection of lot of product. The application of the proposed sampling plan is given for the inspection of gold bumping product and TDT-LCD product. We conclude that the use of the proposed plan in these industries minimizes the cost and time of inspection. Smaller the sample size means low inspection cost. The proposed plan for some non-normal distributions can be extended as a future research. The determination of sampling plan using cost model is also interested area for the future research.

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