Evaluation of Modified Non-Normal Process Capability Index and Its Bootstrap Confidence Intervals

MUHAMMAD KASHIF1,2, MUHAMMAD ASLAM1, ALI HUSSEIN AL-MARSHADI1, CHI-HYUCK JUN3, AND MUHAMMAD IMRAN KHAN2

1Department of Statistics, Faculty of Sciences, King Abdulaziz University, Jeddah 21551, Saudi Arabia
2Department of Mathematics and Statistics, University of Agriculture, Faisalabad 38040, Pakistan
3Department of Industrial and Management Engineering, Pohang University of Science and Technology, Pohang 790-784, South Korea

Corresponding author: Muhammad Kashif (e-mail: mkashif@uaf.edu.pk)

This work was supported in part by the Deanship of Scientific Research and in part by King Abdulaziz University, Jeddah, Saudi Arabia.

ABSTRACT Process capability index (PCI) is used to quantify the process performance and is becoming an attracted area of research. A variability measure plays an important role in PCI. The interquartile range (IQR) or the median absolute deviation (MAD) is commonly used for a variability measure in estimating PCI when a process follows a non-normal distribution. In this paper, the efficacy of the IQR and MAD-based PCIs was evaluated under low, moderate, and high asymmetric behavior of the Weibull distribution using different sample sizes through three different bootstrap confidence intervals. The result reveals that MAD performs better than IQR, because the former produced less bias and mean square error. Also, the percentile bootstrap confidence interval is recommended for use, because it has less average width and high coverage probability.

INDEX TERMS Non-normal distribution, process capability index, interquartile range, median absolute deviation, non-normal, Weibull distribution, bootstrap confidence intervals (BCIs), robust methods.

I. INTRODUCTION

If a process has mean $\mu$ and standard deviation $\sigma$, then the classical process capability index (PCI), $C_p$ is defined as

$$C_p = \frac{USL - LSL}{6\sigma}$$  (1)

where USL and LSL represent the upper and lower specification limits, respectively. The implementation of eq. (1) requires that process should follow a normal distribution [1]. However, in case of engineering and reliability related studies, the assumption of normality is often violated. Therefore, the applicability of the classical PCI may not be appropriate [2]–[4]. During the past few decades the focus has been shifted to the usage of modified non-normal PCIs and their associated properties are also well examined [1], [3]–[7]. Among several approaches [1], [4], [6] to deal with non-normality, the quantiles’ approach [8], [9] is commonly used in practice [2]. The quantile based estimator of the index, $C_p$ requires the replacement of the standard deviation with two quantiles and is defined as

$$C^*_{np} = \frac{USL - LSL}{Q(0.99865) - Q(0.00135)}$$  (2)

where $Q(0.00135)$ and $Q(0.99865)$ are the 0.135th and 99.865th quantiles of the corresponding non-normal distribution, respectively. As pointed out by [10] and [11] that the use of quantile based PCIs, for heavily skewed distributions, did not provide accurate results.

The non-normality have significant influence on the efficiency of the classical PCI defined in eq.(1) because the standard deviation is considered meaningful and efficient measure of variability only for a normal distribution [12]. In case of non-normal distributions, there are other measures that performed better than standard deviation because of their robustness properties. Among those robust measures the commonly used are, median absolute deviation (MAD), interquartile range (IQR), and Gini’s mean difference (GMD). These robust measures are now used in the construction of control charts using different non-normal distributions and showed better performance than existing methods in the
litterature [12, 13]. However, the use of these robust measures in PCIs is not very common.

Rodriguez [14] introduced the idea of robust capability indices and used median and MAD as robust estimators of mean and standard deviation, respectively. Later on, [15] highlighted that the efficiency of MAD and quantile-based estimators of index $C_p$ or $C_{pk}$ were poorer than under Beta, standard normal, student-t and Gamma distributions. This may be because the process capability is affected by the tail behavior of each distribution [4], [16]. Therefore, a method that performed well for a particular distribution may give erroneous results for another distribution with different tail behavior [4].

The Weibull distribution is commonly used for industry oriented processes and has a significantly different tail behavior. But a limited study is available in literature where the performance of MAD based estimator of index $C_p$ has been evaluated. Recently, Besseris [17] introduced distribution free PCIs by replacing traditional location and dispersion parameters with median and interquartile range; and concluded that these robust PCIs performed better than classical PCIs in predicting non-confirming items. Thus, the above reported studies help to conclude that the performance of MAD and IQR for a Weibull distribution is still not fully explored. Therefore, in present study, an effort has been made to compare the performance of two robust capability indices based on MAD and IQR under different asymmetric behavior of Weibull distribution. Moreover, the focus has been made to construct bootstrap confidence intervals for these PCIs.

This research work is an extension of the earlier work [1]. In earlier work [1] only one robust method: Gini’s mean difference (GMD) was applied for study bootstrap confidence interval for two of capability indices $C_p$ and $C_{pk}$. Both studies are collectively helpful to make decision for selecting appropriate capability index for improving industrial process.

The remaining paper is organized as follows. In section II, the PCIs based on IQR and MAD method for Weibull distribution are presented. The results of point and interval estimation of modified indices are explained in section III. A real life example is presented in section IV. Some concluding remarks and recommendations for future studies are discussed in the last section.

II. METHODOLOGY

The Weibull distribution is an important distribution to model process capability related studies [1], [4]. Suppose a random variable $w$ showed exponential distribution with mean $\tau$, then a random variable, $x = w^{1/\nu}$ would be two parameters Weibull distribution with $\nu$ as a shape and $\tau$ as a scale parameter. Then its pdf is given as

$$f(x, \nu, \tau) = \frac{\nu}{\tau} \left(\frac{x}{\tau}\right)^{\nu-1} \exp\left(-\left(\frac{x}{\tau}\right)^{\nu}\right)$$

A. INTER QUARTILE RANGE (IQR)

The inter-quartile range is defined as

$$IQR = Q_3 - Q_1$$

The both upper and lower quantiles are found by solving the following integrals:

$$\int_{-\infty}^{Q_3} f(x) \, dx = 0.75$$

$$\int_{-\infty}^{Q_1} f(x) \, dx = 0.25$$

The $Q_3$ and $Q_1$ for an exponential distribution were $[-\tau \ln(0.25)]$ and $[-\tau \ln(0.75)]$ respectively. Using this, the $Q_3$ and $Q_1$ for the Weibull distribution are given as

$$Q_3 = [-\tau \ln(0.25)]^\frac{1}{\nu}$$

$$Q_1 = [-\tau \ln(0.75)]^\frac{1}{\nu}$$

Therefore,

$$IQR_{\text{weib}} = \left[\frac{\ln(0.25)}{\nu} - \ln(0.75)\right]^\frac{1}{\nu}$$

B. PCIs BASED ON IQR AND MAD

The IQR based estimator of index $C_p$ [17] is given by

$$C_{piw} = \frac{USL - LSL}{2 \times IQR_{\text{weib}}}$$

where the subscript “iw” denotes that the IQR is calculated using Weibull distribution. The formula of $C_p$ using median absolute deviation (MAD) as a measure of variability is defined as [14],

$$C_{pmad} = \frac{USL - LSL}{8.9MAD}$$

where MAD is defined as

$$MAD = b \times \text{median} \{ |x_i - \text{MD}| \} \quad i = 1, 2, \ldots, n$$

The b in (12) is a constant and used for making the parameter of interest as consistent estimator. The term MD in (12) showed the sample median.

C. BOOTSTRAP CONFIDENCE INTERVALS

The commonly used bootstrap confidence intervals (BCIs) are the standard (SB), percentile (PB) and the bias corrected percentile (BCPB) bootstrap method [18]. For these confidence intervals, the bootstrap procedure is explained as follows [19]. Draw a random sample, which consists of $n$ independent and identically distributed random variables $z_1, z_2, z_3, \ldots, z_n$, from the distribution of interest $F$, i.e. $z_1, z_2, z_3, \ldots, z_n \sim F$. Let $\hat{y}$ is the estimator of index $C_p$, which is based on IQR or MAD method. Then,

i. A bootstrap sample, $z_{1}^*, z_{2}^*, \ldots, z_{n}^*$, has been drawn from the original sample with mass of $1/n$ at each point.
II. METHODS

e. If $Z_m^*$ is one of the bootstrap samples, where $(1 \leq m \leq B)$, then its estimator is given as

$$ (y_m^*) = \hat{y}(z_1^*, z_2^*, \ldots, z_m^*) $$

(13)

iii. Each $\hat{y}_m^*$ will be an estimate of $\hat{y}$. The ascending arrangement of all $n^*$ values of the estimator $\hat{y}_m^*$ will make a complete bootstrapped distribution of $\hat{y}$.

The three BCIs of the required PCIs have been constructed; each based on 1000 bootstrapped resample. These BCIs are described below:

1) SB CONFIDENCE INTERVAL

From $B = 1000$, bootstrap estimates of $\hat{\gamma}^*$, calculate the sample average and standard deviation as

$$ \hat{\gamma}^* = (1000)^{-1} \sum_{i=1}^{1000} \hat{\gamma}^*(i) $$

(14)

$$ S^*_{\hat{\gamma}^*} = \sqrt{\left( \frac{1}{999} \right) \sum_{i=1}^{1000} (\hat{\gamma}^*(i) - \hat{\gamma}^*)^2 } $$

(15)

The SB $(1 - \alpha)$ 100% confidence interval is

$$ CI_{SB} = \bar{\gamma}^* \pm Z_{1-\frac{\alpha}{2}} S^*_{\hat{\gamma}^*} $$

(16)

where $Z_{1-\frac{\alpha}{2}}$ is obtained by using $(1 - \frac{\alpha}{2})^{th}$ quantile of the standard normal distribution.

2) PB CONFIDENCE INTERVAL

From the ordered collection of $\hat{\gamma}^*(i)$, choose $100 \left( \frac{\alpha}{2} \right) \%$ and the $100 \left( 1 - \frac{\alpha}{2} \right) \%$ points as the end points to calculate PB. Then, confidence interval would be

$$ CI_{PB} = \left( \hat{\gamma}^*_L, \hat{\gamma}^*_R \right) $$

(17)

For a 95% confidence interval with $B = 1000$, it is:

$$ CI_{PB} = \left( \hat{\gamma}^*_{25}, \hat{\gamma}^*_{975} \right) $$

(18)

3) BCPB CONFIDENCE INTERVAL

The BCPB approach helps to correct the bias. Since the bootstrap distribution is based on a sample drawn from the entire bootstrap distribution, it led to the generation of either upward or downward bias in the estimator. The BCPB interval has been calculated using the following steps:

i. The probability $p_0 = pr \left( \hat{\gamma}^* \leq \hat{\gamma} \right)$ has been computed using the ordered distribution of $\hat{\gamma}^*(i)$.

ii. Computing cumulative and inverse cumulative distribution functions; $\varnothing$ and $\varnothing^{-1}$ of standard normal variable $Z$, i.e.

$$ Z_0 = \varnothing^{-1}(p_0) $$

iii. The lower and upper percentiles of $\hat{\gamma}^*$ are calculated as

$$ P_L = \varnothing \left( 2Z_0 + z_{\frac{1-\alpha}{2}} \right) $$

$$ P_U = \varnothing \left( 2Z_0 + z_{1-\frac{1-\alpha}{2}} \right) $$

The final form of BCPB confidence interval will be

$$ CI_{BCPB} = \left( \hat{\gamma}^*_{(P_L,B)}, \hat{\gamma}^*_{(P_U,B)} \right) $$

(19)

The performance of the three confidence intervals; SB, PB and BCPB were compared using coverage probabilities and average widths. The coverage probability and average width of each BCI are calculated as

$$ Coverage \ Probability = \frac{(L_w \leq C_B \leq U_p)}{B} $$

(20)

$$ Average \ Width = \frac{\sum_{i=1}^{B} (U_p - L_w)}{B} $$

(21)

where $L_w$ and $U_p$ are $(1 - \alpha) \%$ confidence interval based on $B=1000$ replicates.

![Figure 1](image-url)

**FIGURE 1.** The asymmetric behavior of Weibull distribution used for simulation study.

III. RESULTS AND DISCUSSIONS

The point and interval estimation of IQR and MAD-based estimator of index $\hat{C_p}$ using simulation are described in this section. For simulation, different sample sizes i.e. $n = 25, 50, 75 and 100$ and the combination of shape and scale parameters i.e. $[2.8, 3.5], (1.8, 2.0), (1,00, 1.30]$ have been used. These combinations of shape and scale parameters represent the low, moderate and high asymmetric behavior of the distribution and are shown in Figure 1. The lower and upper specification limits were taken as $[0.0, 10.0]$. The mean, standard deviation, bias and RMSE for both PCIs under low, moderate and high asymmetric behavior of Weibull distribution are presented in Table 1. The bias is calculated against the widely used standard target value of $\hat{C_p}$ (equal to 1.33) in the industry which indicates that only 99.73% of the product is within the 75% of the specification limits [20]. The simulation bias and root mean square error (RMSE) are calculated as

$$ Bias = \hat{\gamma} $$

(22)

$$ RMSE = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (y_i - \hat{y})^2 + (Bias)^2} $$

(23)

where $y_i$ represents the value of the estimator with mean $\hat{\gamma}$ and $\gamma$ is the parameter needed to be estimated. The R-software was used for simulation study.
TABLE 1. The mean, standard deviation, bias and root mean square error of the index \(C_p\) using iqr and mad method.

<table>
<thead>
<tr>
<th>(Shape, Scale)</th>
<th>(n=25)</th>
<th>(n=50)</th>
<th>(n=75)</th>
<th>(n=100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (2.8,3.5)</td>
<td>Mean(SD)</td>
<td>RMSE(bias)</td>
<td>Mean(SD)</td>
<td>RMSE(bias)</td>
</tr>
<tr>
<td>n=25</td>
<td>3.5050 (1.0641)</td>
<td>4.7342 (2.1758)</td>
<td>1.5437 (0.4620)</td>
<td>0.0457 (0.2137)</td>
</tr>
<tr>
<td>n=50</td>
<td>3.2233 (0.6346)</td>
<td>3.5845 (1.8933)</td>
<td>1.4301 (0.2820)</td>
<td>0.0100 (0.1001)</td>
</tr>
<tr>
<td>n=75</td>
<td>3.1137 (0.4839)</td>
<td>3.2462 (1.8017)</td>
<td>1.3996 (0.2218)</td>
<td>0.0048 (0.0696)</td>
</tr>
<tr>
<td>n=100</td>
<td>3.0851 (0.4177)</td>
<td>3.0804 (1.7551)</td>
<td>1.3778 (0.1891)</td>
<td>0.0023 (0.0478)</td>
</tr>
<tr>
<td>Moderate (1.8,2.0)</td>
<td>Mean(SD)</td>
<td>RMSE(bias)</td>
<td>Mean(SD)</td>
<td>RMSE(bias)</td>
</tr>
<tr>
<td>n=25</td>
<td>4.2765 (1.3230)</td>
<td>8.6823 (2.9465)</td>
<td>1.9142 (0.5873)</td>
<td>0.3413 (0.5842)</td>
</tr>
<tr>
<td>n=50</td>
<td>3.9050 (0.8048)</td>
<td>6.6309 (2.5750)</td>
<td>1.7673 (0.3616)</td>
<td>0.1912 (0.4373)</td>
</tr>
<tr>
<td>n=75</td>
<td>3.8004 (0.6297)</td>
<td>6.1029 (2.4704)</td>
<td>1.7246 (0.2821)</td>
<td>0.1557 (0.3946)</td>
</tr>
<tr>
<td>n=100</td>
<td>3.7323 (0.5312)</td>
<td>5.7709 (2.4023)</td>
<td>1.6971 (0.2362)</td>
<td>0.1348 (0.3671)</td>
</tr>
<tr>
<td>High (1.0,1.3)</td>
<td>Mean(SD)</td>
<td>RMSE(bias)</td>
<td>Mean(SD)</td>
<td>RMSE(bias)</td>
</tr>
<tr>
<td>n=25</td>
<td>4.3339 (1.7310)</td>
<td>9.0235 (3.0039)</td>
<td>2.1717 (0.8547)</td>
<td>0.7087 (0.8417)</td>
</tr>
<tr>
<td>n=50</td>
<td>3.9019 (1.0326)</td>
<td>6.6149 (2.5719)</td>
<td>1.9705 (0.5146)</td>
<td>0.4102 (0.6405)</td>
</tr>
<tr>
<td>n=75</td>
<td>3.7589 (0.7978)</td>
<td>5.8998 (2.4289)</td>
<td>1.9006 (0.4028)</td>
<td>0.3256 (0.5706)</td>
</tr>
<tr>
<td>n=100</td>
<td>3.6960 (0.6898)</td>
<td>5.5980 (2.3660)</td>
<td>1.8823 (0.3445)</td>
<td>0.3050 (0.5523)</td>
</tr>
</tbody>
</table>

The results of Table 1 showed that both sample size and asymmetric behavior of the Weibull distribution have significant impact on bias and RMSE for IQR and MAD-based estimator of index \(C_p\). As the sample size increases, both bias and RMSE decreases and mean estimated value of indices close to the target value especially in a case of MAD-method. On the other hand, when asymmetric behavior changes from low to high asymmetry, both bias and RMSE increases especially in the case of IQR-method. The performance of MAD-based estimator of index \(C_p\), under low asymmetric behavior, is very accurate. As the sample size increases, the estimated values getting close to the target values (equal to 1.33) and hence a less bias and smaller mean square error have been observed. However, overestimation happened for target value under moderate and high asymmetry. On the other hand, IQR method produced large bias and RMSE as compared to the MAD method under all asymmetric behavior of the distribution even for large sample size. The comparison of bias and RMSE is presented using radar chart in figure 2 and figure 3 respectively. The results of two methods indicate that IQR-method corresponds to worse estimates under all asymmetric levels. On the other hand, MAD gives better estimates using different conditions. MAD is considered a better measure of variability to deal with non-normality while considering bias and RMSE; when process follows the Weibull distribution.

A. INTERVAL ESTIMATION OF MAD-BASED INDEX \(C_p\)

The results of three BCIs are discussed for the only MAD-based estimator of index \(C_p\) because it showed less bias and RMSE. The 95% confidence limits of three methods are presented in Table 2. The coverage probability is reported underneath of each interval. The results revealed that the average width of all confidence intervals reduces when the sample size increases in all cases under study. Moreover, the asymmetric levels affect the average width while the average width increases as asymmetric nature of distribution increases.

From the results of BCIs, the followings conclusions have been drawn.
### TABLE 2. The Bootstrap CIs with coverage probabilities under different asymmetric levels using mad method.

<table>
<thead>
<tr>
<th>n</th>
<th>SB</th>
<th>PB</th>
<th>BCPB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Asymmetry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>(0.7050-2.8914) 0.9990</td>
<td>(1.0249-3.1103) 0.9760</td>
<td>(1.0531-3.3969) 0.6980</td>
</tr>
<tr>
<td>50</td>
<td>(0.9315-2.0352) 0.9990</td>
<td>(1.0391-2.1363) 0.9990</td>
<td>(1.0391-2.1363) 0.7330</td>
</tr>
<tr>
<td>75</td>
<td>(0.8672-1.6262) 0.9990</td>
<td>(0.9157-1.6802) 0.9990</td>
<td>(1.3590-2.0496) 0.7080</td>
</tr>
<tr>
<td>100</td>
<td>(0.1057-1.8643) 0.9990</td>
<td>(1.1401-1.9386) 0.9990</td>
<td>(1.3038-2.2225) 0.7240</td>
</tr>
<tr>
<td></td>
<td>Moderate Asymmetry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>(0.7574-2.9950) 0.9990</td>
<td>(1.0726-3.2716) 0.9810</td>
<td>(1.1690-3.5938) 0.6980</td>
</tr>
<tr>
<td>50</td>
<td>(0.9584-2.0994) 0.9990</td>
<td>(1.0829-2.2358) 0.9990</td>
<td>(1.0737-2.1980) 0.6760</td>
</tr>
<tr>
<td>75</td>
<td>(0.8662-1.6423) 0.9990</td>
<td>(0.9216-1.7134) 0.9950</td>
<td>(1.2119-1.9866) 0.7170</td>
</tr>
<tr>
<td>100</td>
<td>(1.1158-1.9631) 0.9990</td>
<td>(1.1996-2.0112) 0.9990</td>
<td>(1.3566-2.3825) 0.7140</td>
</tr>
<tr>
<td></td>
<td>High Asymmetry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>(0.8642-4.3394) 0.9990</td>
<td>(1.3309-4.6766) 0.9970</td>
<td>(1.4832-5.7500) 0.7270</td>
</tr>
<tr>
<td>50</td>
<td>(1.2653-3.0145) 0.9990</td>
<td>(1.3225-3.1393) 0.9950</td>
<td>(1.2586-3.0536) 0.6070</td>
</tr>
<tr>
<td>75</td>
<td>(0.9834-2.2526) 0.9970</td>
<td>(1.0849-2.3419) 0.9850</td>
<td>(1.2957-2.6615) 0.7160</td>
</tr>
<tr>
<td>100</td>
<td>(1.4297-2.8988) 0.9990</td>
<td>(1.5386-3.0100) 0.9990</td>
<td>(1.7404-3.3987) 0.7350</td>
</tr>
</tbody>
</table>

### TABLE 3. Summary statistics of the data.

<table>
<thead>
<tr>
<th>STATISTIC</th>
<th>VALUE</th>
<th>STATISTIC</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>00.19</td>
<td>AIC&lt;sub&gt;CW&lt;/sub&gt;</td>
<td>140.77</td>
</tr>
<tr>
<td>Max.</td>
<td>72.89</td>
<td>AIC&lt;sub&gt;LS&lt;/sub&gt;</td>
<td>140.82</td>
</tr>
<tr>
<td>Mean</td>
<td>14.36</td>
<td>AIC&lt;sub&gt;GL&lt;/sub&gt;</td>
<td>141.23</td>
</tr>
<tr>
<td>S.d</td>
<td>18.88</td>
<td>BIC&lt;sub&gt;CW&lt;/sub&gt;</td>
<td>142.67</td>
</tr>
<tr>
<td>Q(1)</td>
<td>02.97</td>
<td>BIC&lt;sub&gt;LS&lt;/sub&gt;</td>
<td>142.71</td>
</tr>
<tr>
<td>Q(2)</td>
<td>06.50</td>
<td>BIC&lt;sub&gt;GL&lt;/sub&gt;</td>
<td>143.12</td>
</tr>
<tr>
<td>Q(3)</td>
<td>21.90</td>
<td>Shape</td>
<td>0.7707</td>
</tr>
<tr>
<td>Skewness</td>
<td>01.66</td>
<td>Scale</td>
<td>12.22</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>02.16</td>
<td>USL</td>
<td>75.00</td>
</tr>
<tr>
<td>IQR&lt;sub&gt;50&lt;/sub&gt;</td>
<td>34.20</td>
<td>LSL</td>
<td>0.00</td>
</tr>
<tr>
<td>MAD</td>
<td>05.19</td>
<td>n</td>
<td>19</td>
</tr>
</tbody>
</table>

i. The coverage probability is directly proportional to sample size and reached to the nominal level 0.95 for large sample size in the case of SB and PB method. However, for BCPB method it did not reach to a nominal level, particularly for small samples.

ii. Both BCPB and PB CIs showed less average width as compared to SB. Based on the average with; the three bootstrap methods are ranked as BCPB < PB < SB.

iii. Among BCPB and PB CIs, former showed lower coverage probability than later. Consequently, PB CI performed better for MAD-method.

iv. In all three BCIs, when the transition is made from low to high asymmetric conditions the average width approximately increased by two times. It means under high asymmetry, the width of CI is larger as compared to low and moderate asymmetry.

Based on the low average width and high coverage probability, among three BCIs, the PB CI is recommended under low, moderate and high asymmetric behavior of Weibull process.

### B. EXAMPLE

To test the applicability of the MAD and IQR based index, a numerical examples from industry sector is presented in this section. The data is taken from Nelson, W. [21] and is a part of the data set which contains time to breakdown of an insulating fluid between electrodes records.
TABLE 4. Bootstrap CIs with their coverage probabilities using MAD methods for $C_p$.

<table>
<thead>
<tr>
<th>Method</th>
<th>$C_p$ Value</th>
<th>SB</th>
<th>PB</th>
<th>BCPB</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAD</td>
<td>1.62</td>
<td>(0.5412-4.1855)</td>
<td>0.9990</td>
<td>(0.6067-4.8963)</td>
</tr>
<tr>
<td>IQR</td>
<td>1.09</td>
<td>(0.6745-4.2452)</td>
<td>0.9973</td>
<td>(0.3746-5.2584)</td>
</tr>
</tbody>
</table>

FIGURE 4. Box-Whisker and normal Q-Q plots of data set.

at seven different voltage. Earlier, this data was analyzed by Mukherjee and Singh [22] in order to study the capability of the process. The summary statistics of the data set is reported in Table 3. The Shapiro and Wilks W-test [23] for data is $W = 0.72$ with $p-value = 0.0000$ which indicates that data supports the hypothesis: data follows a non-normal distribution. To confirm that data set follows two parameters the Weibull distribution, the lognormal and gamma distribution is also applied to the data set by using the R-package fitdistrplus [24]. The results are reported in Table 3. The lower value of AIC and BIC showed that Weibull distribution is appropriate as compared to other distributions. The estimates of shape and scale parameters along with upper and lower specification limits are also presented in Table 3.

The box-and-whisker and normal Q-Q plot of the data sets are presented in Figure 4 also supports the above result of W-test. The box-and-whisker plots indicate that data sets have an outlier. Table 4 reports the capability of the two processes using MAD and IQR methods along with three bootstrap confidence intervals. The results of data set under normality showed that the process is not being capable because classical $C_p$ is equal to 0.6620. The normal distribution is not adequate for modeling this data set so the Weibull distribution is used to describe that process. When an adequate model is used; the results of $C_p$ shows that process is being capable. The comparison of proposed $C_p$ values of present study with Mukherjee and Singh [22] ($J = 0.977$) showed that both modified indices give better performance. Furthermore, the MAD based indices are higher than indices based on IQR. The results are not surprising because the data is highly skewed ($Sk ≥ 1.5$) and there is also an outlier in the data set. The three BCIs for this data set are presented in Table 4. The average width and coverage probabilities lead to rank the three BCIs as $PB < SB < BCPB$ in case of MAD-method which also supports the simulation results.

IV. CONCLUSION AND RECOMMENDATIONS

PCIs are important measures for any production process and useful for its continuous improvement. In this study, two robust methods are presented for improving the existing estimation procedures to study the non-normal PCI. The simulation results of point estimation of two methods showed that MAD method is better for the reduction of process variation and yielding high index values. Beside point estimation, interval estimation of MAD-based PCI was constructed because it showed less bias and RMSE. Moreover, three types of bootstrap confidence intervals i.e. $SB$, $PB$, and $BCPB$ and their coverage probabilities using simulation studies were calculated. The selection of the appropriate confidence interval for each method has been made by low average width and higher coverage probability. The simulations illustrated that $PB CI$ is recommended for the MAD-based index $C_p$. Moreover, a real data set example was presented which also support the simulation results. During analysis; the detection of an outlier was also considered while measuring the performance of these indices in the non-normal distributional situation. Numerical outcomes indicate that the proposed indices performed better than existing ones. As a part of future research, the performance of these methods for other advance indices like $C_{pk}$ $C_{pm}$ and $C_{pml}$ using Weibull and other distributions can be considered.

ACKNOWLEDGMENT

The authors are deeply thankful to editor and four reviewers for their valuable suggestions to improve the quality of manuscript. The authors are deeply thankful to editor and reviewers for their valuable suggestions to improve the quality of this manuscript. The authors, therefore, acknowledge with thanks DSR technical support.
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MUHAMMAD KASHIF received the M.Sc. and M.Phil. degrees in statistics from the University of Agriculture, Faisalabad (UAIF), Pakistan. He is currently pursuing the Ph.D. degree with King Abdulaziz University, Jeddah, Saudi Arabia, under its fellowship program. He was a Researcher at various reputable research institutes of Pakistan. He has been working as a Lecturer with the Department of Mathematics and Statistics, UAF, since 2006. He has authored and co-authored over 17 technical papers in applied statistics. His research interests include non-normal distribution, process capability indices, time series analysis, and design of experiments.

MUHAMMAD ASLAM received the M.Sc. degree in statistics from GC University Lahore, with Chief Minister of the Punjab merit scholarship, in 2004, the M. Phil. degree in statistics from GC University Lahore, with the Governor of the Punjab merit scholarship, in 2006, and the Ph.D. degree in statistics from the National College of Business Administration and Economics Lahore under the kind supervision of Prof. Dr. M. Ahmad in 2010. He was a Lecturer of Statistics with the Edge College System International from 2003 to 2006. He was a Research Assistant with the Department of Statistics, GC University Lahore, from 2006 to 2008. Then, he joined the Forman Christian College University as a Lecturer in 2009, where he was an Assistant Professor from 2010 to 2012. He was with the Department of Statistics as an Associate Professor from 2012 to 2014. He supervised five Ph.D. theses, over 25 M.Phil. theses, and three M.Sc. theses. He is currently supervising two Ph.D. theses and over five M.Phil. theses in statistics. He is currently an Associate Professor of Statistics with the Department of Statistics, King Abdul-Aziz University Jeddah, Saudi Arabia. He has published over 220 research papers in national and international journals including for example, the IEEE ACCESS, the Journal of Applied Statistics, the European Journal of Operation Research, the Journal of the Operational Research Society, Applied Mathematical Modeling, the International Journal of Advanced Manufacturer Technology, the Communications in Statistics, the Journal of Testing and Evaluation, and the Pakistan Journal of Statistics. He has authored one book published in Germany. He has been HEC approved Ph.D. supervisor since 2011. He is a Reviewer of more than 45 well reputed international journals. He reviewed more than 75 research papers for various well reputed international journals. His areas of interest include reliability, decision trees, industrial statistics, acceptance sampling, rank set sampling, and applied statistics. Dr. Aslam is a member of Editorial Board of Electronic Journal of Applied Statistical Analysis, Asian Journal of Applied Science and Technology, and Pakistan Journal of Commerce and Social sciences. He is a member of the Islamic Countries Society of Statistical Sciences. He received meritorious services award in research from the National College of Business Administration and Economics Lahore in 2011. He received the Research Productivity Award for the year 2012 by Pakistan Council for Science and Technology. His name was listed at Second Position among Statistician in the Directory of Productivity Scientists of Pakistan 2013. His name was listed at First Position among Statistician in the Directory of Productivity Scientists of Pakistan 2014. He obtained 371 positions in the list of top 2210 profiles of Scientist of Saudi Institutions 2016. He is selected for “Innovative Academic Research and Dedicated Faculty Award 2017” by SPE, Malaysia.
ALI HUSSEIN AL-MARSHADI received the M.S. degree in statistics from New Mexico State University, Las Cruces, NM, USA, in 1997, and the Ph.D. degree in statistics from Oklahoma State University, Stillwater, OK, USA, in 2004. In 1990, he joins the Faculty of Sciences in King Abdul Aziz University, Jeddah, Saudi Arabia, where he is currently a Professor with the Department of Statistics. He has authored and co-authored over 20 technical papers in different areas of statistics. His current research interests include experimental design, linear models, sampling, data mining, and statistical modeling and simulation.

CHI-HYUCK JUN was born in Seoul, South Korea, in 1954. He received the B.S. degree in mineral and petroleum engineering from Seoul National University in 1977, the M.S. degree in industrial engineering from KAIST in 1979, and the Ph.D. degree in operations research from the University of California, Berkeley, in 1986. Since 1987, he has been with the Department of Industrial and Management Engineering, POSTECH, where he is currently a Professor and the Department Head. He is interested in reliability and quality analysis and data mining techniques. Prof. Jun is a member INFORMS and ASQ.

MUHAMMAD IMRAN KHAN was born in Kasur, Pakistan, in 1976. He received the M.Sc. degree in statistics and the M.Phil. degree in statistics from the University of Agriculture, Faisalabad, Pakistan, and the M.S. degree in statistics, the M.S. degree in natural resource sciences, and the Ph.D. degree in natural resource sciences and statistics under Fulbright Scholarship from the University of Nebraska, Lincoln. He has been Teaching and Researching as a Faculty Member with the Department of Mathematics and Statistics, UAF, since 2002. Furthermore, he is developing his expertise to work on simulation studies. His areas of research interest are discrete choice experiments, latent variable models, survey research methods, applied SEE statistics, econometrics, and epidemiology (SSE), program evaluation, policy analysis, and human dimensions of natural resources.

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